3. Groundwater Flow Equations & Well Hydraulics

Groundwater Flow Equations

GW flow eqn. = mathematical expression used to describe the behavior of groundwater flow in porous media.

GW flow eqn = Darcy’s law + Mass Balance eqn.

Darcy’s law:

\[ q_x = \frac{Q}{A} = -K_x \frac{\partial h}{\partial x} \]

In one-dimension:

\[ \bar{q} = \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = - \begin{bmatrix} K_x & 0 & 0 \\ 0 & K_y & 0 \\ 0 & 0 & K_z \end{bmatrix} \begin{bmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \\ \frac{\partial h}{\partial z} \end{bmatrix} \]

Mass balance equation:

\[ \dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{\partial m}{\partial t} \]

Control Volume

\[ \dot{m}_x \]

\[ \Delta x \]

\[ \Delta y \]

\[ \Delta z \]

\[ \dot{m} = \text{mass flow rate entering or exiting a surface} \ [\text{M/T}] \]
\[ \dot{m}_x = \left[ \rho_w q(x) \right] \Delta y \Delta z \]
\[ \dot{m}_y = \left[ \rho_w q(y) \right] \Delta x \Delta z \]
\[ \dot{m}_z = \left[ \rho_w q(z) \right] \Delta x \Delta y \]
\[ \dot{m}_{x+\Delta x} = \left[ \rho_w q(x + \Delta x) \right] \Delta y \Delta z \]
\[ \dot{m}_{y+\Delta y} = \left[ \rho_w q(y + \Delta y) \right] \Delta x \Delta z \]
\[ \dot{m}_{z+\Delta z} = \left[ \rho_w q(z + \Delta z) \right] \Delta x \Delta y \]

\[ \frac{\partial m}{\partial t} = m_{in} - m_{out} = [\dot{m}_x - \dot{m}_{x+\Delta x}] + [\dot{m}_y - \dot{m}_{y+\Delta y}] + [\dot{m}_z - \dot{m}_{z+\Delta z}] \]

\[ \frac{\partial p}{\partial x} \frac{\partial \phi vw}{\partial t} = [\rho_w q_x(x)] \Delta y \Delta z - \left[ \rho_w q_x(x + \Delta x) \right] \Delta y \Delta z \]
\[ + \left[ \rho_w q_y(y) \right] \Delta x \Delta z - \left[ \rho_w q_y(y + \Delta y) \right] \Delta x \Delta z \]
\[ + \left[ \rho_w q_z(z) \right] \Delta x \Delta y - \left[ \rho_w q_z(z + \Delta z) \right] \Delta x \Delta y \]

Substitute porosity \( \phi = \frac{V_w}{V} \) and volume \( V = \Delta x \Delta y \Delta z \) into the above expression

\[ \Delta x \Delta y \Delta z \frac{\partial \phi vw}{\partial t} = [\rho_w q_x(x)] \Delta y \Delta z - \left[ \rho_w q_x(x + \Delta x) \right] \Delta y \Delta z \]
\[ + \left[ \rho_w q_y(y) \right] \Delta x \Delta z - \left[ \rho_w q_y(y + \Delta y) \right] \Delta x \Delta z \]
\[ + \left[ \rho_w q_z(z) \right] \Delta x \Delta y - \left[ \rho_w q_z(z + \Delta z) \right] \Delta x \Delta y \]

Divide through by \( \Delta x \Delta y \Delta z \), we will have

\[ \frac{\partial \phi vw}{\partial t} = \frac{\rho_w q_x(x) - \rho_w q_x(x + \Delta x)}{\Delta x} \]
\[ + \frac{\rho_w q_y(y) - \rho_w q_y(y + \Delta y)}{\Delta y} \]
\[ + \frac{\rho_w q_z(z) - \rho_w q_z(z + \Delta z)}{\Delta z} \]

Assume that density doesn’t change much within the control volume, we can take density out of the left-hand side, then we will have

\[ \frac{1}{\rho_w} \left( \frac{\partial \rho_w}{\partial t} + \frac{\partial \phi}{\partial t} \right) = \frac{q_x(x + \Delta x) - q_x(x)}{\Delta x} - \frac{q_y(y + \Delta y) - q_y(y)}{\Delta y} - \frac{q_z(z + \Delta z) - q_z(z)}{\Delta z} \]

Take limit \( \Delta x, \Delta y, \Delta z \to 0 \), we can transform the difference into the differential form.

Recall the definition of derivative: \( \frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \).
\[
\frac{1}{\rho_w} \left( \frac{\partial \rho_w}{\partial t} + \frac{\partial \phi}{\partial t} \right) = -\lim_{\Delta x \to 0} \left[ \frac{q_x(x + \Delta x) - q_x(x)}{\Delta x} \right] - \lim_{\Delta y \to 0} \left[ \frac{q_y(y + \Delta y) - q_y(y)}{\Delta y} \right] - \lim_{\Delta z \to 0} \left[ \frac{q_z(z + \Delta z) - q_z(z)}{\Delta z} \right]
\]

Finally, we have derived the “general” groundwater flow equation.

\[
(\phi \beta \rho_w g + \alpha \rho_w g) \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left[ K_x \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[ K_y \frac{\partial h}{\partial y} \right] + \frac{\partial}{\partial z} \left[ K_z \frac{\partial h}{\partial z} \right]
\]

For **confined** aquifer: \((\phi \beta \rho_w g + \alpha \rho_w g) = S_x\) or specific storage

\[
S_x \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left[ K_x \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[ K_y \frac{\partial h}{\partial y} \right] + \frac{\partial}{\partial z} \left[ K_z \frac{\partial h}{\partial z} \right]
\]

For **unconfined** aquifer: \((\phi \beta \rho_w g + \alpha \rho_w g) = \frac{S}{b}\) \((S = \text{storativity} = S_x + bS_s)\)

\[
S \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left[ T_x \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[ T_y \frac{\partial h}{\partial y} \right] + \frac{\partial}{\partial z} \left[ T_z \frac{\partial h}{\partial z} \right]
\]

Note: \(T = Kb = \text{transmissivity} \ [L^2/T]\)
\(b = \text{aquifer’s saturated thickness}\)
**Example III-1**

Solve groundwater flow equation in the following 2-D problem by assuming “steady-state” and aquifer is isotropic and homogeneous.

\[
\begin{align*}
\text{Boundary conditions} \\
\text{Top: } h(x, y_0) &= cx + y_0 \quad 0 \leq x \leq s \\
\text{Bottom: } \frac{\partial h}{\partial y} &\bigg|_{y=0} = 0 \quad 0 \leq x \leq s \\
\text{Left: } \frac{\partial h}{\partial x} &\bigg|_{x=0} = 0 \quad 0 \leq y \leq y_0 \\
\text{Right: } \frac{\partial h}{\partial x} &\bigg|_{x=s} = 0 \quad 0 \leq y \leq y_0
\end{align*}
\]

Solve this problem by hand, we will have the solution for \( h(x, y) \) as follows:

\[
h(x, y) = y_0 + \frac{cs}{2} - \frac{4cs}{\pi^2} \sum_{m=0}^{\infty} \cos \left( \frac{2m+1}{2} \frac{\pi x}{s} \right) \cosh \left( \frac{2m+1}{2} \frac{\pi y}{s} \right) \]

***The above example is presented here only for illustration and to show that groundwater flow equation is not easily solved by hand to obtain closed-form solution.***

**Groundwater flow to a pumping well**

Before pumping

While pumping
**Cone of depression**

= dewatered zone in an aquifer (unconfined aquifer)

= a cone of depressed potentiometric surface (confined aquifer)

**Drawdown (s) [L]**

= depressed water level (or potentiometric surface)

**Radius of influence**

= distance from pumping well where drawdown is essentially zero
Pumping in an aquifer with different transmissivity ($T$) and storativity ($S$)

**High $S$**
- We will get more water for the same head drop.

**High $T$**
- Water flows more easily.

**Radial flow to well**

GW flow eqn in cylindrical coordinate can be used to describe radial flow to well.

\[
\frac{S \partial h}{T \partial t} = \frac{1}{r} \frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2}
\]

- $h = \text{hydraulic head [L]}$
- $r = \text{radial distance [L]}$
- $t = \text{time [T]}$

---

Dr. Schradh Saenton

Topic 3: Groundwater Flow and Well Hydraulics
What happens during pumping?

When the well is being pumped, a drawdown of head is created around the well forming a cone of depression. In some cases, if aquifer is being pumped long enough, drawdown can reach equilibrium (i.e. head doesn’t change with time anymore).
Equilibrium flow to well
- Occurs when aquifer is pumped for a very long time.
- Water level (or potentiometric surface) does not change with time.
- We can use darcy’s law to calculate “K” OR “T” if we know Q and hydraulic heads at two locations (i.e. called “pumping test”)

Theim Equation

1. Confined aquifer

\[
T = \frac{Q}{2\pi(h_2 - h_1)} \ln \left( \frac{r_2}{r_1} \right)
\]

\[
K = \frac{Q}{2\pi b(h_2 - h_1)} \ln \left( \frac{r_2}{r_1} \right)
\]

2. Unconfined aquifer

\[
K = \frac{Q}{\pi(h_2^2 - h_1^2)} \ln \left( \frac{r_2}{r_1} \right)
\]

Example III-2
A well in confined aquifer is pumped at a rate of 220 gallon/min. Measurement of drawdown in two observation wells shows that after 1270 min of pumping, no further drawdown is occurring. Well #1 is located at 26 ft from pumping well and has a hydraulic head of 29.34 ft above the top of aquifer. Well #2 is located at 73 ft from the pumping well and has a hydraulic head of 32.56 ft above the top of aquifer. Use Thiem equation to find aquifer transmissivity if aquifer thickness is 25 m.
Steady-state (equilibrium) pumping in an unconfined aquifer

(Example of model simulation)

No Pumping Condition

Convert \( Q \) from gal/min to \( \text{ft}^3/\text{day} \)

\[
Q = \left[ \frac{220 \text{ gal}}{\text{min}} \right] \left[ \frac{1 \text{ ft}^3}{7.48 \text{ gal}} \right] \left[ \frac{1440 \text{ min}}{1 \text{ day}} \right]
\]

\[
= 42,400 \text{ ft}^3/\text{day}
\]

\[
T = \frac{Q}{2\pi(h_2 - h_1) \ln \left[ \frac{r_2}{r_1} \right]}
\]

\[
= \frac{42400 \text{ ft}^3/\text{day}}{2\pi(32.56 \text{ ft} - 29.34 \text{ ft}) \ln \left[ \frac{73 \text{ ft}}{26 \text{ ft}} \right]}
\]

\[
= 2164 \text{ ft}^2/\text{day}
\]

\[
T = Kb \rightarrow 2164 \frac{\text{ft}^2}{\text{d}} = K[78 \text{ ft}] \rightarrow K = 27.7 \text{ ft/day}
\]
With pumping at the center of the aquifer

Non-equilibrium pumping (or transient)

Drawdown \( s \) is a function of time and distance

**Confined aquifer**

- Use Theis solution
- Aquifer is homogeneous, isotropic, and is of infinite extent
- Well completely penetrates (and get water from) the entire aquifer
- Transmissivity is constant
- Water is removed from storage and discharge instantaneously.
Theis Solution

\[ s = h_0 - h = \frac{Q}{4\pi T} W(u) \]

where \( u = \frac{r^2 S}{4Tt} \), and \( W(u) = \int_0^\infty \frac{e^{-x}}{x} \, dx = -0.5772 - \ln u + \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} + \cdots \)

- \( s \) drawdown [L]
- \( h_0 \) initial head in a well at distance \( r \) [L]
- \( h \) head at distance \( r \) at time \( t \) [L]
- \( t \) time since pumping begins [T]
- \( r \) distance from pumping well [L]
- \( Q \) pumping rate [L³/T]
- \( T \) transmissivity [L²/T]
- \( S \) storativity [-]
- \( b \) aquifer’s saturated thickness [L]
- \( W(u) \) well function [-]
- \( u \) auxiliary parameter [-]

**Example III-3**

A well is located in confined aquifer with a hydraulic conductivity of 14.9 m/day and a storativity of 0.0051. The aquifer is 20.1 m thick and is pumped at a rate of 2725 m³/day.

What is the drawdown at a distance of 7.0 m from the well after 1 day of pumping?

\[
T = Kb = \left[ 14.9 \text{ m/day} \right] (20.1 \text{ m}) = 299.49 \text{ m}^2/\text{day}
\]

\[
u = \frac{r^2 S}{4Tt} = \frac{[7 \text{ m}]^2 \times [0.0051]}{4 \times 299.49 \text{ m}^2/\text{day} \times 1 \text{ day}} = 2.086 \times 10^{-4}
\]

Find the value of \( W(u) \) from the table (appendix 1)
Using “linear interpolation technique”

\[
x - 7.94 = \frac{2.086 \times 10^{-4} - 2.0 \times 10^{-4}}{7.53 - 7.94} \times 3.0 \times 10^{-4} - 2.0 \times 10^{-4}
\]

\[
x = 7.9398
\]

From table + linear interpolation \( W(u) = 7.9398 \). Therefore the drawdown at distance of 7.0 m after 1 day of pumping is

\[
s = h_0 - h = \frac{Q}{4\pi T} W(u) = \frac{2725 \text{ m}^3/\text{day}}{4\pi \times 299.49 \text{ m}^3/\text{day}} \times 7.9398 = 5.75 \text{ m}
\]

**Unconfined aquifer**

- Use Neuman solution
- Aquifer is homogeneous and is of infinite extent
- Initially water is pumped from storage \((S_i)\)
- Later, water is being drained due to gravity \((S_y)\)
- Assume drawdown is negligible compared to saturated thickness
- Radial \(K\) (or \(K_r\)) can be different from vertical \(K\) (or \(K_v\))
- *Neuman solution is valid only when drawdown is very small compared to aquifer’s thickness or* \(s << b\)
Neuman solution

\[ s = h_0 - h = \frac{Q}{4\pi T} W(u_A, u_B, \Gamma) \]

Where \( u_A = \frac{r^2 S}{4Tt} \), \( u_B = \frac{r^2 S}{4Tt} \) and \( \Gamma = \frac{r^2 K_v}{b^2 K_h} \)

\( W(u_A, u_B, \Gamma) \) can be obtained from table (appendix 6A, 6B)

\[ s \] drawdown [L]
\[ h_0 \] initial head in a well at distance \( r \) [L]
\[ h \] HEAD AT DISTANCE \( r \) AT TIME \( t \) [L]
\[ t \] time since pumping begins [T]
\[ r \] distance from pumping well [L]
\[ Q \] pumping rate \([L^3/T]\)
\[ K_v \] vertical hydraulic conductivity \([L/T]\)
\[ K_h \] horizontal hydraulic conductivity \([L/T]\)
\[ S \] storativity at early time (in this case, \( S = bS_s \)) \([-\)]
\[ S_v \] specific yield \([-\)]
\[ b \] initial aquifer’s saturated thickness [L]

At early pumping time, use \( u_A \)

Where at later time, use \( u_B \).
**Example III-4**

A well is located in an unconfined aquifer with a vertical and horizontal hydraulic conductivities of 1.26 and 15.8 m/day, respectively. The value of specific storage is 0.00025 m\(^{-1}\), and specific yield is 0.12. The aquifer’s initial saturated thickness is 20.1 m and is pumped at a rate of 275 m\(^3\)/day. What is the drawdown at a distance of 7.0 m from the well after 1 and 50 day of pumping?

\[ r = 7 \text{ m} \quad Q = 275 \text{ m}^3/\text{day} \]
\[ K_v = 1.26 \text{ m/day} \quad K_h = 15.3 \text{ m/day} \]
\[ b = 20.1 \text{ m} \quad S_v = 0.12 \]
\[ S = bS_v = (0.00025 \text{ m}^{-1}) \times (20.1 \text{ m}) = 0.005 \]
\[ T = K_hb = (15.8 \text{ m/day}) \times (20.1 \text{ m}) = 317.58 \text{ m}^2/\text{day} \]

\[ \Gamma = \frac{r^2K_v}{b^2K_h} = \frac{[7 \text{ m}]^2 \times [1.26 \text{ m/day}]}{[20.1 \text{ m}]^2 \times [15.3 \text{ m/day}]} \approx 0.01 \]  
[see table in appendix 6A, 6B]

**After 1 day (early time)**

\[ u_a = \frac{r^2S}{4\pi t} = \frac{[7 \text{ m}]^2 \times [0.005]}{4 \times [317.58 \text{ m}^2/\text{day}][1 \text{ day}]} = 1.93 \times 10^{-4} \rightarrow 1/u_a = 5185 \rightarrow W(u) = 3.46 \]

Drawdown: \[ s = \frac{Q}{4\pi T}W(u) = \frac{[275 \text{ m}^3/\text{day}]}{4\pi [317.58 \text{ m}^2/\text{day}]} \times 3.46 = 0.238 \text{ m} \]

**After 50 day (late time)**

\[ u_a = \frac{r^2S}{4\pi t} = \frac{[7 \text{ m}]^2 \times [0.12]}{4 \times [317.58 \text{ m}^2/\text{day}][50 \text{ day}]} = 9.26 \times 10^{-5} \rightarrow 1/u_a = 10802 \rightarrow W(u) = 8.672 \]

Drawdown: \[ s = \frac{Q}{4\pi T}W(u) = \frac{[275 \text{ m}^3/\text{day}]}{4\pi [317.58 \text{ m}^2/\text{day}]} \times 8.672 = 0.598 \text{ m} \]

**Check:** \[ s = 0.238, 0.597 \ll b=20.1 \ldots \ldots \text{OK!!} \]
**Summary: Characteristics of Analyses**

<table>
<thead>
<tr>
<th>Equilibrium (or steady-state)</th>
<th>Non-Equilibrium (or transient)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Obtained most accurate $T,K$ values</td>
<td>1. Can determine storativity (in pumping test)</td>
</tr>
<tr>
<td>2. Useful when long-term pumping has been established</td>
<td>2. Get results at early time</td>
</tr>
<tr>
<td>3. Equations useful in designing a pump test (estimate maximum drawdown)</td>
<td>3. Analysis is more complicated</td>
</tr>
<tr>
<td>4. Cannot obtain information on storage</td>
<td>4. Theis solution $\rightarrow$ confined</td>
</tr>
<tr>
<td>5. Thiem solution</td>
<td>5. Neuman solution $\rightarrow$ unconfined</td>
</tr>
<tr>
<td>6. Neuman solution is applicable only for $s \ll b$</td>
<td></td>
</tr>
</tbody>
</table>

**Principle of superposition**

- If there are more than one pumping wells, drawdown at observation well can be determined using *principle of superposition* (drawdown can be added or subtracted)
- Normally, this method is valid **only** for “confined aquifer.” However, in some cases, it may be applicable in unconfined aquifer if drawdown is negligible compared to saturated thickness ($s \ll b$).
**Example III-5**

Two wells in a confined aquifer \((b = 20 \text{ m}, S = 0.0075, K = 1.75 \text{ m/day})\) are pumping simultaneously at the rates of 200 and 400 \(\text{m}^3/\text{day}\), respectively. Calculate drawdown at an observation well at \(t = 4 \text{ day}\).

Using principle of superposition: \(s_{total} = s_{pw1} + s_{pw2}\)

1. **calculate drawdown at OW from PW1**
   \[ T = Kb = (1.75 \text{ m/day}) \times (20 \text{ m}) = 35 \text{ m}^2/\text{day} \]
   \[ u = \frac{r^2S}{4Tt} = \frac{(97 \text{ m})^2 \times 0.0075}{4 \times 35 \text{ m}^2/\text{day} \times 4 \text{ day}} = 0.126 \rightarrow W(u) = 1.667 \]
   \[ s_{pw1} = \frac{Q}{4\pi T} W(u) = \frac{200 \text{ m}^3/\text{day}}{4\pi [35 \text{ m}^2/\text{day} \times 4 \text{ day}]} \times 1.667 = 0.758 \text{ m} \]

2. **calculate drawdown at OW from PW2**
   \[ u = \frac{r^2S}{4Tt} = \frac{(175 \text{ m})^2 \times 0.0075}{4 \times 35 \text{ m}^2/\text{day} \times 4 \text{ day}} = 0.410 \rightarrow W(u) = 0.688 \]
   \[ s_{pw2} = \frac{Q}{4\pi T} W(u) = \frac{400 \text{ m}^3/\text{day}}{4\pi [35 \text{ m}^2/\text{day}]} \times 0.688 = 0.626 \text{ m} \]

**Example III-6**

From Example III-5, if, instead of pumping water from PW2, water is injected at the rate of +400 \(\text{m}^3/\text{day}\) while PW1 is still pumping at the rate of -200 \(\text{m}^3/\text{day}\). Calculate drawdown in observation well at time \(t = 4 \text{ day}\).
From previous example,

\[ s_{pw1} = \text{drawdown due to pumping from PW1 (water level decreases)} = +0.758 \text{ m} \]

\[ s_{injw2} = \text{drawup due to injection of water in PW2 (water level increases)} = -0.626 \text{ m} \]

From principle of superposition, drawdown at observation well after day 4 is

\[ s_{total} = s_{pw1} + s_{injw2} = 0.758 - 0.626 = 0.13 \text{ m} \] Thus, water level decreases only 0.13 m in this case.

**Method of Images**

When an aquifer is connected to river or impermeable barrier, pumping well will be affected by these boundaries. One can use method of image to calculate “correct” drawdown at the observation well.

- If boundary is river (water supply) → an image well will be an “injection” well.
- If boundary is impermeable rock → an image well will be a “pumping” well.

Once an image well is created, one can use principle of superposition to determine accurate drawdown from the pumping/injecting wells.
If boundary is river, image well of a pumping well is an injection well. On the other hand, if the well is an injection well, image will be an injection well. If boundary is an impermeable rock, image of the pumping well is a pumping well where image of an injection well is an injection well.

<table>
<thead>
<tr>
<th>River Boundary</th>
<th>Impermeable Boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well</td>
<td>Image</td>
</tr>
<tr>
<td>pumping</td>
<td>injection</td>
</tr>
<tr>
<td>injection</td>
<td>Injection</td>
</tr>
</tbody>
</table>

**Example III-7**

An aquifer is begin pumped from PW1 (pumping well #1). If this aquifer is connected to an impermeable (no flow) boundary, suggest the method how to calculate drawdown in an observation well (OW).

Since the boundary is impermeable, an image well should be a “pumping well” and drawdown at OW can be calculated from “PW1” and “PW1-img” as shown below.
Principle of superposition:

\[ s_{\text{total}} = s_{\text{PW}_1} + s_{\text{PW}_{1-IMG}} \]

\[ = \frac{Q}{4\pi T} W(u_{\text{PW}_1}) + \frac{Q}{4\pi T} W(u_{\text{PW}_{1-IMG}}) \]

\[ = \frac{Q}{4\pi T} \left[ W(u_{\text{PW}_1}) + W(u_{\text{PW}_{1-IMG}}) \right] \]

where \( u_{\text{PW}_1} = \frac{r^2 S}{4Tt} \) and \( u_{\text{PW}_{1-IMG}} = \frac{r_2^2 S}{4Tt} \)

**Seawater intrusion**

- In a coastal aquifer, freshwater usually overlies on top of saline groundwater due to density difference
- Saline groundwater can move landward if too much pumping occurs.
- Boundary between fresh and saline groundwater is not sharp! There is a zone of diffusion where the concentration (of salt) gradient exists.

**Ghyben-Berzberg Principle**

In an unconfined aquifer, the relationship between water table and depth to saline groundwater is described by the following expression:

\[
  z(x, y) = \left[ \frac{\rho_{\text{fresh}}}{\rho_{\text{saline}} - \rho_{\text{fresh}}} \right] h(x, y)
\]

where

- \( z(x, y) \) depth of salt-water interface below sea level (L)
- \( h(x, y) \) elevation of water table above sea level (L)
- \( \rho_{\text{fresh}} \) density of fresh groundwater (M/L^3)
- \( \rho_{\text{saline}} \) density of saline groundwater (M/L^3)
Example III-8

If density of fresh water and saline groundwater are 1000 and 1025 kg/m³, respectively, what is the ratio of \( \frac{z(x,y)}{h(x,y)} \)?

\[
\frac{z(x,y)}{h(x,y)} = \frac{1000}{1025 - 1000} h(x,y)
\]

\[
\frac{z(x,y)}{h(x,y)} = 40
\]

Thus, depth to which fresh groundwater extends below sea level is approximately 40 times the height of water table above sea level.
Exercise

1. A well that pumps at a constant rate of 78,000 ft$^3$/day has achieved equilibrium so that there is no change in the drawdown with time. The well taps a confined aquifer that is 18 ft thick. An observation well 125 ft away has a head of 277 ft above sea level; another observation well 385 ft away has a head of 291 ft. Compute the value of aquifer transmissivity using Thiem equation.

   Ans: 997.5 ft$^2$/day

2. A well that pumps at a constant rate of 78,000 ft$^3$/day has achieved equilibrium so that there is no change in the drawdown with time. The well taps an unconfined aquifer that consists of sand overlying impermeable bedrock at an elevation of 260 ft above sea level. An observation well 125 ft away has a head of 277 ft above sea level; another observation well 385 ft away has a head of 291 ft. Compute the value of hydraulic conductivity using Thiem equation.

   Ans: 41.6 ft/day

3. A community is installing a new well in a regionally confined aquifer with a transmissivity of 1589 ft$^2$/day and a storativity of 0.0005. The planned pumping rate is 325 gal/min. There are several nearby wells tapping the same aquifer, and the project manager needs to know if the new well will cause significant interference with these wells. Compute the theoretical drawdown caused by the new well after 30 days of continuous pumping at the following distances: 50, 150, 250, 500, 1000, 3000, 6000, and 10,000 ft.

   Ans: 35.56, 28.70, 25.58, 21.14, 16.85, 6.87, 5.87, and 3.18 ft

4. A well that is screened in a confined aquifer is to be pumped at a rate of 165,000 ft$^3$/day for 30 days. If the aquifer transmissivity is 5320 ft$^2$/day, and the storativity is 0.0007, what is the drawdown at distances of 50, 150, 250, 500, 1000, 3000, 5000, and 10,000 ft?

5. A well is being pumped from an unconfined aquifer that has initial saturated thickness of 30 m. This aquifer has similar vertical and horizontal conductivities (i.e., $K_v = K_h = 10$ m/day) with $S_v = 0.0001$ m$^{-1}$ and $S_h = 0.2$. Calculate drawdown at observation well, located at 5.477 m away from the pumping well, at time $t = 1$ day (early time) and $t = 50$ day (late time). Use $Q = 100$ m$^3$/day.

6. A well (PW1) is pumping water from confined aquifer, that is close to an impermeable rock, at a rate of 100 m$^3$/day. Calculate drawdown at observation well (OW) at time $t = 5$ d. Given $S = 0.0005$, $T = 500$ m$^3$/day.
7. A well (PW1) is pumping water from confined aquifer, that is close to a fully penetrated river, at a rate of 150 m$^3$/day. Calculate drawdown at observation well (OW) at time $t = 10$ d. Given $S = 0.0005$, $T = 500$ m$^2$/day.